

D-brane worlds and the cosmological constant

P. Gusin, J. Warczewski

University of Silesia, Institute of Physics, ul. Uniwersytecka 4,
PL-40007 Katowice, Poland
e-mail: pgusin@us.edu.pl

Abstract

The cosmological constant on a D-brane is analyzed. This D-brane is in the background produced by the p-brane solutions. The energy-momentum tensor in this model has been found and the form of the cosmological constant has been derived. This energy-momentum tensor is interpreted as an energy-momentum tensor for a perfect fluid on the D-brane. The energy density and the pressure for this fluid have been derived. As it turned out the pressure is negative but the speed of sound is real.

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1 Introduction

The problem of the cosmological constant, interpreted as a vacuum energy, consists in that the vacuum energy obtained from the general relativity (GR) equations is much smaller, then the vacuum energy obtained from the particle theory (standard model). This discrepancy can be overcome if one chooses the initial conditions with the highest accuracy. This leads to the so called fine-tuning problem. In order to solve these problems several models have been proposed. One of these consists in the modification of the GR on the distances bigger than size of the present universe [1]. The extra dimensions, in this model, have to remain infinite (non-compact) in order to get consistent theory. From the other side the fine-tuning problem is solved by the statistical approach to the different vacua of the superstring theory with the compactified extra dimensions [2]. Each vacuum realizes a 4-dimensional particle theory with a hidden sector. Parameters of this sector determinate, among others, the vacuum energy of the 4-dimensional universe. Thus in the huge number of the superstring vacua some part of them can realize the observed small value of the cosmological constant. The p-brane solutions of the low-energetic supergravity in the type IIA/IIB string theory and the discovery of the D-branes in the open string theory give new view on the cosmological models ([3], [4]). These branes are extended and interact with each other by gravity. Each brane interacts also with itself.

We consider an energy-momentum tensor induced on the D-brane by the non-trivial background given by p-brane solutions. This tensor projected on the D-brane world-volume has an interpretation of the cosmological constant. We present an explicit form of the cosmological constant as a function of the transverse directions to the D-brane.

2 Gravity generated by p -branes

The form of the gravity when the fundamental constituents of matter are p -branes is considered e.g. in [3], [4], [5] and [10].

Let us recall the form of an action and the solutions for the system consisting of a dilaton ϕ , a graviton g_{MN} and an antisymmetric tensor $A_{M_1\dots M_d}$ of arbitrary rank d in a D dimensional space-time R^D coupled to an extended object.

The action $I_D(d)$ for $\phi, g_{MN}, A_{M_1\dots M_d}$ has the form [10]:

$$I_D(d) = 1/2\kappa^2 \int_{R^D} d^D x \sqrt{-g} (R(g) - 1/2 |d\phi|^2 - \frac{1}{2(d+1)!} e^{-\phi a(d)} F^2), \quad (2.1)$$

where:

$$F^2 = F_{M_1\dots M_{d+1}} F^{M_1\dots M_{d+1}}, \\ F_{M_1\dots M_{d+1}} = (dA)_{M_1\dots M_{d+1}}.$$

The above fields are coupled to an elementary d -dimensional extended object $((d-1)$ -brane) M with a world-volume metric $\gamma_{\mu\nu}$. This brane is embedded into R^D :

$$X : M \rightarrow R^D.$$

An action S_d for this brane is given by:

$$S_d = T_d \int_{M \times \mathbf{R}} d^d \xi \left[-\frac{1}{2} \sqrt{-\gamma} \gamma^{\mu\nu} \partial_\mu X^M \partial_\nu X^N g_{MN} e^{a\phi/d} + \right. \\ \left. \frac{d-2}{2} \sqrt{-\gamma} \right. \\ \left. - \frac{1}{d!} \varepsilon^{\mu_1\dots\mu_d} \partial_{\mu_1} X^{M_1} \dots \partial_{\mu_d} X^{M_d} A_{M_1\dots M_d} \right], \quad (2.2)$$

where $\mu, \nu = 0, 1, \dots, d-1$. Thus the action $I(D, d)$ for the system consists of the sum of the actions (2.1) and (2.2):

$$I(D, d) = I_D(d) + S_d. \quad (2.3)$$

In the action (2.3) there are five independent fields:

1. an antisymmetric field $A_{M_1\dots M_d}$,
2. a metric g_{MN} on R^D ,

3. a dilaton field ϕ ,
4. a vector field X which makes an embedding of the brane M into R^D ,
5. a metric $\gamma_{\mu\nu}$ on M .

The equations of motion with the respect the above fields are:

- The condition $\frac{\delta I(D,d)}{\delta A} = 0$ gives (the Maxwell equations with the sources):

$$d * (e^{-a\phi} F) = 2\kappa^2 * J, \quad (2.4)$$

where the current J is given by:

$$J^{M_1 \dots M_d}(x) = T_d \int_{M \times \mathbf{R}} d^d \xi \varepsilon^{\mu_1 \dots \mu_d} \partial_{\mu_1} X^{M_1} \dots \partial_{\mu_d} X^{M_d} \chi, \quad (2.5)$$

and $\chi = \delta^D(x - X(\xi)) / \sqrt{-g}$

- The Einstein equations $\frac{\delta I(D,d)}{\delta g} = 0$:

$$\begin{aligned} R_{MN} - \frac{1}{2} g_{MN} R = & \frac{1}{2} \left(\partial_M \phi \partial_N \phi - \frac{1}{2} g_{MN} |d\phi|^2 \right) \\ & + \kappa^2 T_{MN} \\ & + \frac{e^{-a\phi}}{2d!} \left(F_M^{M_1 \dots M_d} F_{N M_1 \dots M_d} - \frac{1}{2(d+1)} g_{MN} F^2 \right), \end{aligned} \quad (2.6)$$

where $T_{MN} = g_{MA} g_{NB} T^{AB}$ is the energy-momentum tensor of the brane M :

$$T^{AB}(x) = T_d \int_{M \times \mathbf{R}} d^d \xi \sqrt{-\gamma} \gamma^{\mu\nu} \partial_\mu X^A \partial_\nu X^B e^{a\phi/d} \chi, \quad (2.7)$$

and $\chi = \delta^D(x - X(\xi)) / \sqrt{-g}$

- The dilaton equation $\frac{\delta I(D,d)}{\delta \phi} = 0$:

$$\begin{aligned} \partial_M (\sqrt{-g} g^{MN} \partial_N \phi) + \frac{a\sqrt{-g}}{2(d+1)} e^{-a\phi} F^2 = \\ = -\kappa^2 \frac{a}{d} T_d \int_M d^d \xi \sqrt{-\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N g_{MN} e^{a\phi/d} \chi \sqrt{-g}. \end{aligned} \quad (2.8)$$

- The brane equations $\frac{\delta I(D,d)}{\delta X} = 0$:

$$\begin{aligned} \partial_\mu \left(\sqrt{-\gamma} \gamma^{\mu\nu} \partial_\nu X^N g_{MN} e^{a\phi/d} \right) + \\ - \frac{1}{2} \sqrt{-\gamma} \gamma^{\mu\nu} \partial_\mu X^N \partial_\nu X^P \partial_M \left(g_{NP} e^{a\phi/d} \right) = \\ \frac{1}{d!} \varepsilon^{\mu_1 \dots \mu_d} \partial_{\mu_1} X^{M_1} \dots \partial_{\mu_d} X^{M_d} F_{M M_1 \dots M_d}. \end{aligned} \quad (2.9)$$

- The equations of motion $\frac{\delta I(D,d)}{\delta \gamma} = 0$ for the world metric γ :

$$\gamma_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N g_{MN} e^{a\phi/d}. \quad (2.10)$$

In order to solve the above coupled system of equations (2.4-2.9), it is assumed that R^D has the topology of the Cartesian product [10]:

$$R^D = M \times N, \quad (2.11)$$

where M is a d -dimensional manifold ($(d-1)$ -brane) with the Poincare symmetry group $P(d)$ and N is an isotropic manifold with $SO(D-d)$ symmetry group. The coordinates on R^D are split:

$$X^M = (x^\mu, y^m),$$

where x^μ , y^m concerns M and N , respectively. The indices μ and m have the range: $\mu = 0, 1, \dots, d-1$; $m = 1, \dots, D-d$. In this topology the ansatz for the metric g_{MN} has the form:

$$ds^2 = g_{MN} dx^M dx^N = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{2B(y)} \delta_{mn} dy^m dy^n, \quad (2.12)$$

the metric η is diagonal: $(\eta_{\mu\nu}) = \text{diag}(+1, -1, \dots, -1)$. The functions A and B depend only on $y = (\mathbf{y} \cdot \mathbf{y})^{1/2}$. The form of the antisymmetric field A is assumed below:

$$A_{\mu_1 \dots \mu_d} = -\frac{1}{\det(g_{\mu\nu})} \varepsilon_{\mu_1 \dots \mu_d} e^{C(y)}, \quad (2.13)$$

where:

$$\varepsilon_{\mu_1 \dots \mu_d} = g_{\mu_1 \nu_1} \dots g_{\mu_d \nu_d} \varepsilon^{\nu_1 \dots \nu_d},$$

($\varepsilon^{01 \dots d-1} = +1$) and the other components of A are set to zero, $\det(g_{\mu\nu}) = (-1)^{d-1} e^{2Ad}$. Thus the field F has the form:

$$F_{m\mu_1 \dots \mu_d} = -\frac{1}{\det(g_{\mu\nu})} \varepsilon_{\mu_1 \dots \mu_d} \partial_m (e^{C(y)}). \quad (2.14)$$

The dilaton field ϕ depends on y since N is isotropic:

$$\phi = \phi(y).$$

A static gauge choice for the vector field X is also assumed:

$$X^\mu = \xi^\mu,$$

where ξ^μ are coordinates on the brane M . In this static gauge the field X is equal to:

$$X^M = (\xi^\mu, Y^m).$$

The directions Y transverse to the brane M are constant: $Y^m = \text{const.}$ It means that the brane is not moving in this special coordinates system. Under the above conditions the metric γ (Eq. (2.10)) takes the form:

$$\gamma_{\mu\nu} = \eta_{\mu\nu} e^{2A+a\phi/d}.$$

One of the solutions for the above system with the flat asymptotic condition ($g_{MN} \rightarrow \eta_{MN}$) is given by [10]:

$$A(y) = \frac{\tilde{d}}{2(d+\tilde{d})} (C(y) - C_0), \quad (2.15)$$

$$B(y) = -\frac{d}{2(d+\tilde{d})} (C(y) - C_0), \quad (2.16)$$

$$e^{-C(y)} = \begin{cases} e^{-C_0 + \frac{k_d}{y^d}} & \text{for } \tilde{d} > 0 \\ e^{-C_0 + \frac{\kappa^2 T_d}{\pi} \ln y} & \text{for } \tilde{d} = 0 \end{cases}, \quad (2.17)$$

$$\frac{a}{d}\phi(y) = \frac{a^2}{4} (C(y) - C_0) + C_0, \quad (2.18)$$

$$a^2(d) = 4 - \frac{2d\tilde{d}}{d+\tilde{d}}, \quad (2.19)$$

where:

$$k_d = \frac{2\kappa^2 T_d}{\Omega_{\tilde{d}+1} \tilde{d}} \quad (2.20)$$

and $\tilde{d} = D - d - 2$, $\Omega_{\tilde{d}+1}$ is the volume of a $(\tilde{d} + 1)$ -dimensional sphere $S^{\tilde{d}+1}$. Thus the metric g_{MN} is given by:

$$\begin{aligned} g_{MN} dX^M dX^N &= \left(1 + \frac{k_d}{y^d} e^{-C_0}\right)^{-\frac{\tilde{d}}{d+\tilde{d}}} \eta_{\mu\nu} dx^\mu dx^\nu + \\ &\left(1 + \frac{k_d}{y^d} e^{-C_0}\right)^{\frac{d}{d+\tilde{d}}} \delta_{mn} dy^m dy^n. \end{aligned} \quad (2.21)$$

The other solution for this system is given by a $(d + 2)$ -dimensional black-brane with the symmetry group:

$$\mathbf{R} \times SO(d+1) \times SO(\tilde{d}-1).$$

The metric for this system has the form:

$$\begin{aligned} ds^2 &= -\Delta_+ \Delta_-^{-\frac{\tilde{d}}{d+\tilde{d}}} dt^2 + \Delta_+^{-1} \Delta_-^{\frac{a^2}{2d}-1} dr^2 + r^2 \Delta_-^{\frac{a^2}{2d}} d\Omega_{d+1}^2 \\ &+ \Delta_-^{\frac{d}{d+\tilde{d}}} dX_i dX^i, \end{aligned} \quad (2.22)$$

where $i = 1, \dots, \tilde{d} - 1$ and:

$$e^{-2\phi} = \Delta_-^a, \quad (2.23)$$

$$\Delta_{\pm} = 1 - \left(\frac{r_{\pm}}{r}\right)^d, \quad (2.24)$$

$$F_{d+1} = (r_+ r_-)^{d/2} \varepsilon_{d+1} d, \quad (2.25)$$

and ε_{d+1} is the volume form of the $(d+1)$ -dimensional sphere S^{d+1} with the metric $d\Omega_{d+1}^2 = h_{ab} d\varphi^a d\varphi^b$. The radii r_+ and r_- are related to the mass $M_{\tilde{d}}$ per unit $(\tilde{d}-1)$ -volume and to the magnetic charge $g_{\tilde{d}}$:

$$M_{\tilde{d}} = \int d^{D-d} \Theta_{00} = \frac{\Omega_{d+1}}{2\kappa^2} [(d+1) r_+^d - r_-^d], \quad (2.26)$$

$$g_{\tilde{d}} = \frac{1}{\sqrt{2\kappa}} \int_{S^{d+1}} e^{-a\phi} * F = \frac{\Omega_{d+1}}{\sqrt{2\kappa}} d(r_+ r_-)^{d/2}, \quad (2.27)$$

where Θ_{MN} is the total energy-momentum tensor for the system and $*$ is the Hodge duality operator with respect to the metric (2.22). In the case when $r_+ = r_- = r_0$ the mass and charge are given by:

$$M_{\tilde{d}} = \sqrt{2\kappa} g_{\tilde{d}}. \quad (2.28)$$

It means that this brane becomes the extremal p-brane (BPS state). In this case the metric (2.22) takes the form:

$$ds^2 = \Delta^{\frac{d}{d+d}} (-dt^2 + dX_i dX^i) + \Delta^{-\frac{\tilde{d}}{d+d}} (d\rho^2 + \rho^2 d\Omega_{d+1}^2), \quad (2.29)$$

where $\rho^d = r^d - r_0^d$ and $\Delta = 1 + (r_0/\rho)^d$.

3 A D-brane motion in the field of the black-brane

We consider a $D_{d'-1}$ -brane M embedded in the background of the $(d+2)$ -blackbrane N in the D -dimensional space-time R^D . This $(d+2)$ -blackbrane wraps a $(d+1)$ -dimensional sphere. The metric of R^D in the presence of black-brane is given by Eq. (2.22). Thus the metric $\gamma_{\alpha\beta}$ induced on M by g_{MN} has the form:

$$\gamma_{\alpha\beta} = g_{MN} \frac{\partial X^M}{\partial \xi^\alpha} \frac{\partial X^N}{\partial \xi^\beta}, \quad (3.1)$$

where X is an embedding of M in R^D :

$$\begin{aligned} X &: N \times \mathbf{R}^1 \rightarrow R^D, \\ X^M &= X^M(\xi^0, \xi^a) \end{aligned}$$

and $\alpha, \beta = 0, 1, \dots, d' - 1$, $n = 1, \dots, d' - 1$. We assume that the time in R^D and in the worldvolume M is the same and $d' - 1$ directions of N are parallel to M . Thus the embedding of X^M has the form:

$$X^M(\xi^0, \xi^a) = (\xi^0, \xi^a, X^m(\xi^0)), \quad (3.2)$$

where $a = 1, \dots, d' - 1$ and $m = 1, \dots, D - d'$. The coordinates on M and R^D selected in this way form the static gauge. For the metric g_{MN} (which is produced by (d+2)-brane wrapped on S^{d+1}) equal to

$$ds^2 = \lambda_0 dt^2 + \lambda_1 \sum_{i=1}^{\tilde{d}-1} dX_i^2 + \lambda_2 dr^2 + r^2 \lambda_3 d\Omega_{d+1} \quad (3.3)$$

the metric $\gamma_{\alpha\beta}$ induced by the embedding (3.2) takes the form:

$$\gamma_{00} = \lambda_0 + \lambda_1 \sum_{i=d'}^{\tilde{d}-1} \dot{X}_i^2 + \lambda_2 \dot{r}^2 + r^2 \lambda_3 \dot{\varphi}^2, \quad (3.4)$$

$$\gamma_{ab} = \lambda_1 \delta_{ab}, \text{ for } d' - 1 \leq \tilde{d} - 1 \quad (3.5)$$

and $\gamma_{a0} = 0$, where:

$$\dot{\varphi}^2 = h_{rs} \dot{\varphi}^r \dot{\varphi}^s,$$

and $h_{rs} = h_{rs}(\varphi)$ ($r, s = 1, \dots, d+1$) is the metric on S^{d+1} . The coordinates X^m in the metric (3.3) are as follows:

$$X^m = (X^i, r, \varphi^s),$$

where $i = d', \dots, \tilde{d} - 1$. In the case when the metric of the background has the form:

$$ds^2 = \lambda_0 dt^2 + \lambda_1 \sum_{i=1}^{\tilde{d}-1} dX_i^2 + \lambda_2 \sum_{m=1}^{d+2} dX_m^2, \quad (3.6)$$

the induced metric takes the form:

$$\gamma_{00} = \lambda_0 + \lambda_1 \sum_{i=d'}^{\tilde{d}-1} \dot{X}_i^2 + \lambda_2 \sum_{m=1}^{d+2} \dot{X}_m^2, \quad (3.7)$$

$$\gamma_{0a} = 0, \quad (3.8)$$

$$\gamma_{ab} = \lambda_1 \delta_{ab}, \text{ for } d' - 1 \leq \tilde{d} - 1. \quad (3.9)$$

If $d' - 1 \geq \tilde{d} - 1$ the metric γ is given by:

$$\gamma_{00} = \lambda_0 + \lambda_2 \sum_{m=1}^{d+2} \dot{X}_m^2, \quad (3.10)$$

$$\gamma_{a_1 b_1} = \lambda_1 \delta_{a_1 b_1} \text{ for } a_1, b_1 = 1, \dots, \tilde{d} - 1, \quad (3.11a)$$

$$\gamma_{a_2 b_2} = \lambda_2 \delta_{a_2 b_2} \text{ for } a_2, b_2 = \tilde{d}, \dots, d' - 1. \quad (3.11b)$$

Thus in the gauge (3.2) the metric γ induced on M by the blackbrane N (the latter producing the background metric (2.27)) has the form:

$$\gamma_{00} = -\Delta_+ \Delta_-^{-\frac{\tilde{d}}{d+d}} + \Delta_+^{-1} \Delta_-^{\frac{a^2}{2d}-1} \dot{r}^2 + r^2 \Delta_-^{\frac{a^2}{2d}} \dot{\varphi}^2 + \Delta_-^{\frac{d}{d+d}} \dot{X}_i \dot{X}^i, \quad (3.12)$$

$$\gamma_{0a} = 0, \quad (3.13)$$

$$\gamma_{ab} = \Delta_-^{\frac{d}{d+d}} \delta_{ab}. \quad (3.14)$$

For the static case $\dot{X}_i = 0$ ($i = 1, \dots, \tilde{d} - 1$) γ_{00} takes the form:

$$\gamma_{00} = -\Delta_+ \Delta_-^{-\frac{\tilde{d}}{d+d}} + \Delta_+^{-1} \Delta_-^{\frac{a^2}{2d}-1} \dot{r}^2 + r^2 \Delta_-^{\frac{a^2}{2d}} \dot{\varphi}^2. \quad (3.15)$$

4 The energy-momentum tensor for a D-brane

The energy-momentum tensor of the $(d' - 1)$ -brane M in the background g_{MN} is given by Eq. (2.7) and is expressed by the matrix:

$$(T^{MN}) = \begin{pmatrix} T^{00} & T^{0a} & T^{0m} \\ T^{a0} & T^{ab} & T^{am} \\ T^{m0} & T^{ma} & T^{mn} \end{pmatrix}, \quad (4.1)$$

where $a, b = 1, \dots, d' - 1$ and $m, n = 1, \dots, D - d'$. The generic form of the background metric g_{MN} produced by a $(d - 1)$ -brane is given by (see (3.3) and (3.6)):

$$(g_{MN}) = \begin{pmatrix} \begin{pmatrix} \lambda_0 & (0) \\ (0) & \lambda_1 I_{\tilde{d}-1} \end{pmatrix} & 0 \\ 0 & (g_{rs}) \end{pmatrix}, \quad (4.2)$$

where $I_{\tilde{d}-1}$ is $(\tilde{d} - 1)$ -dimensional unit matrix and $r, s = 1, \dots, d + 2$. Thus the induced metric $\gamma_{\mu\nu}$ on the brane M for the embedding (3.2) has the form given either by Eqs.(3.4-3.5) or by Eqs.(3.7-3.11). The components of the energy-momentum tensor T^{MN} for the metric (4.2) in the embedding (3.2) take the form:

$$T^{\mu\nu} = T_d \sqrt{\frac{\gamma}{g}} \gamma^{\mu\nu} e^{a\phi/d} \widehat{\delta}, \quad (4.3)$$

$$T^{m0} = T^{0m} = T_d \sqrt{\frac{\gamma}{g}} \gamma^{00} \dot{X}^m e^{a\phi/d} \widehat{\delta}, \quad (4.4)$$

$$T^{mn} = T_d \sqrt{\frac{\gamma}{g}} \gamma^{00} \dot{X}^m \dot{X}^n e^{a\phi/d} \widehat{\delta}, \quad (4.5)$$

where $\widehat{\delta} = \delta^{D-d} (x^m - X^m(\xi^0))$. Other components (T^{a0}, T^{am}) are equal to zero and $\gamma = \det(\gamma_{\mu\nu})$, $g = \det(g_{MN})$.

4.1 Cosmological constant induced by the blackbranes

The ratio of the determinants γ/g for the metrics (3.3), (3.4) and (3.5) is given by:

$$\gamma/g = \frac{\lambda_1^{d'-\tilde{d}}\Gamma}{r^{2(d+1)}\lambda_2\lambda_3^{d+1}\det h}, \quad (4.6)$$

where:

$$\Gamma = 1 + \frac{\lambda_1}{\lambda_0} \sum_{i=d'}^{\tilde{d}-1} \dot{X}_i^2 + \frac{\lambda_2}{\lambda_0} r^2 + r^2 \frac{\lambda_3}{\lambda_0} \dot{\varphi}^2, \quad (4.7)$$

and $\det h$ is the determinant of the metric h_{rs} on the sphere S^{d+1} . For the embedding (3.2) and in the metric (3.3) the time-dependent components \dot{X}^m in Eqs.(4.4-4.5) have the form:

$$\dot{X}^m = \left(\dot{X}^i, \dot{r}, \dot{\varphi}^s \right),$$

where $i = d', \dots, \tilde{d} - 1$ and $s = 1, \dots, d + 1$ (φ^s are coordinates on S^{d+1}). In this way we obtain the explicit form of T^{MN} :

$$T^{\mu\nu} = \frac{T_{d'}}{r^{d+1}} \sqrt{\frac{\lambda_1^{d'-\tilde{d}}\Gamma}{\lambda_2\lambda_3^{d+1}\det h}} \gamma^{\mu\nu} e^{a\phi/d} \hat{\delta}, \quad (4.8)$$

$$T^{m0} = \frac{T_{d'}}{r^{d+1}} \sqrt{\frac{\lambda_1^{d'-\tilde{d}}\Gamma}{\lambda_2\lambda_3^{d+1}\det h}} \frac{\dot{X}^m}{\lambda_0\Gamma} e^{a\phi/d} \hat{\delta}, \quad (4.9)$$

$$T^{mn} = \frac{T_{d'}}{r^{d+1}} \sqrt{\frac{\lambda_1^{d'-\tilde{d}}\Gamma}{\lambda_2\lambda_3^{d+1}\det h}} \frac{\dot{X}^m \dot{X}^n}{\lambda_0\Gamma} e^{a\phi/d} \hat{\delta}, \quad (4.10)$$

since $\gamma_{00} = \lambda_0\Gamma$ and $\gamma^{00} = (\lambda_0\Gamma)^{-1}$. The D -brane tension $T_{d'}$ is given by [12, 13]:

$$T_{d'}^2 = \frac{\pi}{\kappa_{(10)}^2} (4\pi^2\alpha')^{4-d'}.$$

The pull-back of the T^{MN} by the embedding X gives the energy-momentum tensor $\tilde{T}_{\mu\nu}$ on the D-brane:

$$\tilde{T}_{\mu\nu} = T^{AB} g_{AM} g_{BN} \frac{\partial X^M}{\partial \xi^\mu} \frac{\partial X^N}{\partial \xi^\nu}. \quad (4.11)$$

Thus we obtain from (4.11):

$$\begin{aligned} \tilde{T}_{00} &= T^{00} g_{00}^2 + T^{m_1 n_1} g_{m_1 m} g_{n_1 n} \dot{X}^m \dot{X}^n \\ \tilde{T}_{0a} &= 0, \\ \tilde{T}_{ab} &= T^{cd} g_{ca} g_{bd}, \end{aligned}$$

where

$$\begin{aligned} g_{00} &= \lambda_0, \\ (g_{m_1 m}) &= \begin{pmatrix} \lambda_1 I_{\tilde{d}-d'} & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & r^2 \lambda_3 (h_{ps}) \end{pmatrix}, \\ (g_{ac}) &= \lambda_1 I_{d'-1}. \end{aligned}$$

Because $g_{mn} \dot{X}^m \dot{X}^n = \lambda_0 (\Gamma - 1)$ we get from (4.8-10):

$$\tilde{T}_{00} = \frac{T_{d'}}{r^{d+1}} \sqrt{\frac{\lambda_1^{d'-\tilde{d}}}{\lambda_2 \lambda_3^{d+1} \Gamma \det h}} e^{a\phi/d} \lambda_0 [\Gamma^2 - 2\Gamma + 2], \quad (4.12)$$

$$\tilde{T}_{ab} = \frac{T_{d'}}{r^{d+1}} \sqrt{\frac{\lambda_1^{d'-\tilde{d}} \Gamma}{\lambda_2 \lambda_3^{d+1} \det h}} e^{a\phi/d} \lambda_1 \delta_{ab}, \quad (4.13)$$

modulo delta functions. In the static case ($\dot{X}^m = 0$) $\Gamma = 1$, so the Eqs. (4.12-13) take the form:

$$\tilde{T}_{\mu\nu} = \frac{T_{d'}}{r^{d+1}} \sqrt{\frac{\lambda_1^{d'-\tilde{d}}}{\lambda_2 \lambda_3^{d+1} \det h}} e^{a\phi/d} \gamma_{\mu\nu}. \quad (4.14)$$

This tensor consists of the part

$$\Lambda_b(r; d', d) = \frac{T_{d'}}{r^{d+1}} \sqrt{\frac{\lambda_1^{d'-\tilde{d}}}{\lambda_2 \lambda_3^{d+1} \det h}} e^{a\phi/d}, \quad (4.15)$$

which depends only on the direction r transverse to the brane M . Thus for a fixed position of M in the ambient space R the quantity Λ_b has the constant value. The equations of gravity on M take the form:

$$R_{\mu\nu} - \frac{1}{2} \gamma_{\mu\nu} R = t_{\mu\nu} + \tilde{T}_{\mu\nu}, \quad (4.16)$$

where $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature with respect to the metric $\gamma_{\mu\nu}$ and $t_{\mu\nu}$ is the energy-momentum tensor for the matter and fields on the D-brane. Because $\tilde{T}_{\mu\nu}$ is the product of Λ_b (which is constant on M) and the metric $\gamma_{\mu\nu}$, the equation (4.16) takes the form:

$$R_{\mu\nu} - \frac{1}{2} \gamma_{\mu\nu} R = t_{\mu\nu} + \Lambda_b \gamma_{\mu\nu}.$$

Thus Λ_b can be identified as a cosmological constant which is produced by the other branes. For the metric (2.22) one obtains that:

$$\Lambda_b(r; d', d) = \frac{T_{d'}}{r^{2(d+1)}} \sqrt{\frac{\Delta_+}{\det h}} \Delta_-^\sigma, \quad (4.17)$$

where:

$$\sigma(d', d; D) = \frac{\tilde{d}(3-d) + d'd}{2(D-2)} - \frac{3}{\tilde{d}} + \frac{1}{2}. \quad (4.18)$$

In the static case induced by the $(d-1)$ -dimensional blackbrane the term Λ_b on the $D_{d'-1}$ -brane takes the form:

$$\Lambda_b(r; d', d) = \frac{T_{d'}}{r^{2(d+1)}} \sqrt{\frac{1}{\det h}} \left(1 - \frac{r_+^d}{r^d}\right)^{1/2} \left(1 - \frac{r_-^d}{r^d}\right)^\sigma, \quad (4.19)$$

where the radial coordinate r is interpreted as a distance from the center of the blackbrane wrapped on S^{d+1} to the center of the $(d'-1)$ -brane. The dimensions of the blackbranes change from 0 to $D-1$. Thus the total term Λ_b induced by the set of blackbranes of different dimensions can be expressed by the following sum:

$$\Lambda_b(r_1, \dots, r_{D-1}; d') = \sum_{d=1}^{D-1} \Lambda_b(r_d; d', d), \quad (4.20)$$

where r_d is the distance from $(d-1)$ -dimensional brane to $(d'-1)$ -brane.

In the case, when $D = 10$ and $d' = 4$,

$$\sigma(4, d; 10) = 2 - \frac{3}{d} + \frac{1}{16}(d-7)d. \quad (4.21)$$

Thus

$$\Lambda_b(r; 4, d) = \frac{T_4}{r^{2(d+1)}} \sqrt{\frac{1}{\det h}} \left(1 - \frac{r_+^d}{r^d}\right)^{1/2} \left(1 - \frac{r_-^d}{r^d}\right)^{2 - \frac{3}{d} + \frac{1}{16}(d-7)d} \quad (4.22)$$

and the total cosmological constant is given by:

$$\Lambda_b(r_1, \dots, r_9; 4) = \sum_{d=1}^9 \Lambda_b(r_d; 4, d). \quad (4.23)$$

In this way we showed that the induced cosmological constant is the function of dimensions of the blackbranes and distances from them to the 4-dimensional brane.

In the non-static case ($\Gamma \neq 1$) we introduce a scalar field ϕ which is related to the transverse coordinates of the blackbrane:

$$\phi^2 = \frac{\lambda_1}{|\lambda_0|} \sum_{i=d'}^{\tilde{d}-1} \dot{X}_i^2 + \frac{\lambda_2}{|\lambda_0|} \dot{r}^2 + r^2 \frac{\lambda_3}{|\lambda_0|} \dot{\varphi}^2. \quad (4.24)$$

Thus:

$$\Gamma = 1 - \phi^2, \quad (4.25)$$

since λ_0 is negative. The Eqs. (4.12-13) take the forms:

$$\tilde{T}_{00} = \Lambda_b \frac{1 + \phi^4}{\sqrt{1 - \phi^2}} \gamma_{00}, \quad (4.26)$$

$$\tilde{T}_{ab} = \Lambda_b \sqrt{1 - \phi^2} \gamma_{ab}, \quad (4.27)$$

where Λ_b is given by (4.19) or by (4.22) for $d' = 4$. Let us compare in the commoving frame ($u_a = 0$) this induced energy-momentum tensor to an energy-momentum tensor for a perfect fluid $T_{\mu\nu}$ with an energy density ε and a pressure p :

$$T_{\mu\nu} = (\varepsilon + p) u_\mu u_\nu - p \gamma_{\mu\nu}. \quad (4.28)$$

As a result of this comparison one obtains:

$$\varepsilon = \Lambda_b \frac{1 + \phi^4}{\sqrt{1 - \phi^2}}, \quad (4.29)$$

$$p = -\Lambda_b \sqrt{1 - \phi^2}. \quad (4.30)$$

The corresponding state equation has the form:

$$w = p/\varepsilon = -\frac{1 - \phi^2}{1 + \phi^4}. \quad (4.31)$$

For the variety of the blackbranes we get a set of the fields ϕ_d . Thus the effective energy and the pressure have the form:

$$\varepsilon = \sum_{d=1}^9 \Lambda_b(r_d; 4, d) \frac{1 + \phi_d^4}{\sqrt{1 - \phi_d^2}}, \quad (4.32)$$

$$p = -\sum_{d=1}^9 \Lambda_b(r_d; 4, d) \sqrt{1 - \phi_d^2}. \quad (4.33)$$

In this case the state equation is:

$$w = -\frac{\sum_{d=1}^9 \Lambda_b(r_d; 4, d) \sqrt{1 - \phi_d^2}}{\sum_{d=1}^9 \Lambda_b(r_d; 4, d) \frac{1 + \phi_d^4}{\sqrt{1 - \phi_d^2}}}. \quad (4.34)$$

One can see from above that for the certain values of the fields ϕ_d the state equation assumes the form $w \leq -1/3$ which corresponds to the exotic matter interpretation on the D3-brane.

4.2 Cosmological constant induced by the branes without horizon

In this case the background metric is given by (2.21) and the induced metric γ is given by (3.7-9) for $d' - 1 \leq \tilde{d} - 1$ and by (3.10-11a,b) for $d' - 1 \geq \tilde{d} - 1$. Thus

the ratio of the corresponding determinants has the form:

$$\frac{\det \gamma}{\det g} = \begin{cases} \frac{\lambda_1^{d'-\tilde{d}}}{\lambda_2^{d+2}} \Gamma & \text{for } d' - 1 \leq \tilde{d} - 1, \\ \lambda_2^{d'+2d+4-D} \Omega & \text{for } d' - 1 \geq \tilde{d} - 1, \end{cases} \quad (4.35)$$

where:

$$\Gamma = 1 + \frac{\lambda_1}{\lambda_0} \sum_{i=d'}^{\tilde{d}-1} y_i^{\cdot 2} + \frac{\lambda_2}{\lambda_0} \sum_{m=1}^{d+2} y_m^{\cdot 2}, \quad (4.36)$$

$$\Omega = 1 + \frac{\lambda_2}{\lambda_0} \sum_{m=1}^{d+2} y_m^{\cdot 2}. \quad (4.37)$$

Let us consider first the case when $d' - 1 \leq \tilde{d} - 1$. Thus:

$$T^{\mu\nu} = T_{d'} \sqrt{\frac{\lambda_1^{d'-\tilde{d}} \Gamma}{\lambda_2^{d+1}}} \gamma^{\mu\nu} e^{a\phi/d} \widehat{\delta}, \quad (4.38)$$

$$T^{m0} = T_{d'} \sqrt{\frac{\lambda_1^{d'-\tilde{d}} \Gamma}{\lambda_2^{d+1}}} \gamma^{00} y^{\cdot m} e^{a\phi/d} \widehat{\delta}, \quad (4.39)$$

$$T^{mn} = T_{d'} \sqrt{\frac{\lambda_1^{d'-\tilde{d}} \Gamma}{\lambda_2^{d+1}}} \gamma^{00} y^{\cdot m} y^{\cdot n} e^{a\phi/d} \widehat{\delta}. \quad (4.40)$$

Proceeding as before one obtains the following pull-back of this tensor on the $D_{d'-1}$ -brane:

$$\tilde{T}_{00} = T^{00} g_{00}^2 + T^{m_1 n_1} g_{m_1 m} g_{n_1 n} y^{\cdot m} y^{\cdot n} \quad (4.41)$$

$$\tilde{T}_{0a} = 0, \quad (4.42)$$

$$\tilde{T}_{ab} = T^{cd} g_{ca} g_{bd}, \quad (4.43)$$

where:

$$g_{00} = \lambda_0, \quad (4.44)$$

$$(g_{m_1 m}) = \begin{pmatrix} \lambda_1 I_{\tilde{d}-d'} & \\ & \lambda_2 I_{d+2} \end{pmatrix}, \quad (4.45)$$

$$(g_{ac}) = \lambda_1 I_{d'-1}. \quad (4.46)$$

Thus:

$$\tilde{T}_{00} = T_{d'} \sqrt{\frac{\lambda_1^{d'-\tilde{d}}}{\lambda_2^{d+1} \Gamma}} e^{a\phi/d} \lambda_0 [\Gamma^2 - 2\Gamma + 2], \quad (4.47)$$

$$\tilde{T}_{ab} = T_{d'} \sqrt{\frac{\lambda_1^{d'-\tilde{d}} \Gamma}{\lambda_2^{d+1}}} e^{a\phi/d} \lambda_1 \delta_{ab}. \quad (4.48)$$

In the static case ($\Gamma = 1$) this energy-momentum tensor has the form:

$$\tilde{T}_{\mu\nu} = \Lambda'_o \gamma_{\mu\nu}, \quad (4.49)$$

where:

$$\Lambda'_o(d', d, D; y) = T_{d'} \left(1 + \frac{k_d}{y^{\tilde{d}}} e^{-C_0} \right)^\sigma \quad (4.50)$$

and

$$\sigma = \frac{d}{2(d + \tilde{d})} (d' + 2\tilde{d} - 2). \quad (4.51)$$

In the superstring regime $d + \tilde{d} = 8$ and for $d' = 4$ the exponent σ is equal to:

$$\sigma = \frac{d(18-d)}{16} \quad (4.52)$$

and $d \leq 4$. Thus the term Λ'_o takes the form:

$$\Lambda'_o(4, d, 10; y) = T_4 \left(1 + \frac{k_d}{y^{\tilde{d}}} e^{-C_0} \right)^{\frac{d(18-d)}{16}} \quad (4.53)$$

and for $y \rightarrow \infty$ tends to T_4 .

The second case is for $d' - 1 \geq \tilde{d} - 1$. The induced energy-momentum tensor is given by:

$$\tilde{T}_{00} = T_{d'} \sqrt{\frac{\lambda_2^{d'+2d+4-D}}{\Omega}} e^{a\phi/d} \lambda_0 [\Omega^2 - 2\Omega + 2], \quad (4.54)$$

$$\tilde{T}_{ab} = T_{d'} \sqrt{\lambda_2^{d'+2d+4-D}} \Omega e^{a\phi/d} \lambda_1 \delta_{ab}. \quad (4.55)$$

Thus as before in the static case ($\Omega = 1$) we obtain:

$$\tilde{T}_{\mu\nu} = \Lambda''_o \gamma_{\mu\nu}, \quad (4.56)$$

where:

$$\Lambda''_o(d', d, D; y) = T_{d'} \left(1 + \frac{k_d}{y^{\tilde{d}}} e^{-C_0} \right)^{-\sigma}, \quad (4.57)$$

and

$$\sigma = \frac{\tilde{d}}{2(d + \tilde{d})} (D + d' + d). \quad (4.58)$$

In the superstring regime $d + \tilde{d} = 8$ and for $d' = 4$ the exponent σ is equal to:

$$\sigma = \frac{(d+14)(10-d)}{16} \quad (4.59)$$

and $d > 4$. Thus the term Λ_o'' takes the form:

$$\Lambda_o''(4, d, 10; y) = T_4 \left(1 + \frac{k_d}{y^d} e^{-C_0} \right)^{-\frac{(d+14)(10-d)}{16}}. \quad (4.60)$$

In the presence of the variety of branes with different dimensions the total induced cosmological constant is given by:

$$\Lambda(y_1, \dots, y_9) = \sum_{d=1}^4 \Lambda_o'(4, d, 10; y_d) + \sum_{d=5}^9 \Lambda_o''(4, d, 10; y_d). \quad (4.61)$$

In the non-static case we introduce as before a field ϕ defined by:

$$\phi^2 = \frac{\lambda_1}{|\lambda_0|} \sum_{i=d'}^{\tilde{d}-1} y_i^2 + \frac{\lambda_2}{|\lambda_0|} \sum_{m=1}^{d+2} y_m^2. \quad (4.62)$$

Thus:

$$\tilde{T}_{00} = \Lambda \frac{1 + \phi^4}{\sqrt{1 - \phi^2}} \gamma_{00}, \quad (4.63)$$

$$\tilde{T}_{ab} = \Lambda \sqrt{1 - \phi^2} \gamma_{ab}, \quad (4.64)$$

where Λ is expressed by (4.53) for $d \leq 4$ and by (4.60) for $d > 4$. Proceeding as at the end of the previous section one gets energy, pressure and the state equation for the exotic matter induced by a non-blackbrane on D3-brane:

$$\varepsilon = \Lambda \frac{1 + \phi^4}{\sqrt{1 - \phi^2}}, \quad (4.65)$$

$$p = -\Lambda \sqrt{1 - \phi^2}, \quad (4.66)$$

$$w = p/\varepsilon = -\frac{1 - \phi^2}{1 + \phi^4}, \quad (4.67)$$

where Λ is the same as in (4.63). For the variety of branes one gets:

$$\varepsilon = \sum_{d=1}^4 \Lambda_o'(r_d; 4, d) \frac{1 + \phi_d^4}{\sqrt{1 - \phi_d^2}} + \sum_{d=5}^9 \Lambda_o''(r_d; 4, d) \frac{1 + \phi_d^4}{\sqrt{1 - \phi_d^2}}, \quad (4.68)$$

$$p = -\sum_{d=1}^4 \Lambda_o'(r_d; 4, d) \sqrt{1 - \phi_d^2} - \sum_{d=5}^9 \Lambda_o''(r_d; 4, d) \sqrt{1 - \phi_d^2}. \quad (4.69)$$

In this case the state equation is:

$$w = -\frac{\sum_{d=1}^4 \Lambda_o'(r_d; 4, d) \sqrt{1 - \phi_d^2} + \sum_{d=5}^9 \Lambda_o''(r_d; 4, d) \sqrt{1 - \phi_d^2}}{\sum_{d=1}^4 \Lambda_o'(r_d; 4, d) \frac{1 + \phi_d^4}{\sqrt{1 - \phi_d^2}} + \sum_{d=5}^9 \Lambda_o''(r_d; 4, d) \frac{1 + \phi_d^4}{\sqrt{1 - \phi_d^2}}}. \quad (4.70)$$

4.3 The cosmological constant

In order to get the total induced cosmological constant on the D3-brane we should take into account all kinds of p-branes with different dimensions and with different distances from their centers to the center of D3-brane. Thus collecting the results (4.23) and (4.61) one obtains:

$$\Lambda = \sum_{d=1}^9 \Lambda_b(4, d, 10; r_d) + \sum_{d=1}^4 \Lambda'_o(4, d, 10; y_d) + \sum_{d=5}^9 \Lambda''_o(4, d, 10; y_d). \quad (4.71)$$

The dynamic case is described on the D3-brane by a set of fields $\{\phi_S\}$ (where $S = 0, 1, \dots, 9$ is the dimension of the p-brane in the 10-dimensional ambient space).

The global energy-momentum tensor for the perfect fluid on the D3-brane is produced by the configurations of branes of different kinds in 10-dimensional ambient time-space. Thus the cosmological constant induced on the D3-brane can be fitted to the observed one by the appropriate choice of the parameters appearing in (4.71). As it is well-known the evolution of the universe depends on the value of the cosmological constant. In the present time the universe is accelerated. This phenomenon is being usually explained by an assumption of the existence of the exotic matter described by the state equation $w < -1/3$. Such an exotic matter produces negative pressure, which acts against gravitation. Our approach presented above allows us to obtain such a state equation by the appropriate choice of the values of certain parameters in (4.71). The Eq. (4.71) changes with time which means that the evolution of D3-brane presented above depends not only on the D3-brane contents but also on the p-branes configuration in the ambient space.

5 Conclusions

The form of the cosmological constant Λ on the D3-brane M has been derived as a pull-back of the energy-momentum tensor, the latter tensor being taken for the background produced by the different p-branes. The contributions coming from the gravity solutions for the p-branes have been taken into account only and both the gauge fields on M and RR charges of the p-branes have been ignored. In this way the dependence of the cosmological constant on both the dimensions of the p-branes and their distances to M has been obtained.

In the dynamic case when $\phi \neq 0$ one obtains the energy-momentum tensor on M which can be identified with the energy-momentum tensor for the perfect fluid on M . This perfect fluid representing some kind of the exotic matter has the state equations given either by Eq.(4.34) in the case of the blackbranes or by Eq. (4.70) in the case of the p-branes without horizon. The energy and pressure are given by (4.32) and (4.33) for the first case as well as by (4.68) and (4.69) for the second case. The pressure produced by this perfect fluid is negative and acts against gravitation. One can then say that the perfect fluid can present one of the factors determining the cosmological evolution of M . Thus the evolution

of M depends on its position in the ambient space with respect to the other branes. As one can see the speed of sound c_s is real as a function of the field ϕ for $\phi \in (-1, 1)$ (here only one brane is considered):

$$c_s = \sqrt{\frac{dp}{d\varepsilon}} = \sqrt{\frac{1 - \phi^2}{1 + 4\phi^2 + \phi^3 - 3\phi^4}},$$

where ε is given by the Eq. (4.29) and p is given by the Eq. (4.30). The picture below shows the ϕ -dependence of c_s :



The model presented above can be interpreted as one of the models for the explanation of the origin of the dark energy. In this model the dark energy is generated by the perfect fluid of the exotic matter. In [14] the dark energy is associated with p-branes in the light-cone parametrization.

6 References

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